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Aerodynamic Simulation of a FSAE Open-Wheel Car During Cornering

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Abstract

This work develops and applies a steady Cornering Simulation Model (CSM) for the external aerodynamics of a Formula SAE (FSAE) open-wheel car. The approach replaces conventional Straight-Line Simulation Models (SLSM) by solving the Reynolds-averaged Navier–Stokes equations in a Rotating Reference Frame (RRF) representative of a constant-radius turn. The formulation introduces the Coriolis and centrifugal source terms and enforces curvature-consistent kinematics at the inlet, road, and wheels. A simplified 2-DoF bicycle vehicle model supplies the steering, sideslip, ride attitude, and wheel speeds consistent with the selected lateral acceleration and turn radius. The method pursues cornering for aerodynamic design while acknowledging quasi-steady limitations. An example skidpad-based case illustrates the workflow from tyre modeling to CFD setup.

NB : this paper is a proper re-edition of the collaborative work involving Eline Houri, for her Final Year Project (MGA961) at ÉTS, and Hugues Perrin, for his Research Project (TX51) at UTBM. References about the literature review [1] and straight line simulation [2] are part of this re-edition.

Introduction

Since around 2013, Formule ÉTS has been developing their aerodynamic devices using CFD methods. Although poor relevancy to typical use-cases of aerodynamic forces, it has led to bold and efficient designs recognized worldwide by other FSAE or Formula Student teams. The poor relevancy of a SLSM comes from two major factors : generating negative lift is mostly used to increase the grip of a car during cornering, resulting in a higher theoretical lateral acceleration ; most of the time on a regular FSAE or Formula Student track is spent cornering (i.e a steering angle above 20 degrees).

Preparing a cornering case comes with a few challenges, both in the aerodynamical modeling and the vehicle dynamics modeling. In particular, accurately capturing the transient flow structures and the localized variations in pressure distribution over the aerodynamic surfaces could require an unsteady simulation approach, or a very precise and well converged steady state hypothesis, typically employing high-fidelity models with sufficiently refined grids in regions of high curvature and vortex formation.

From a vehicle dynamics standpoint, setting up a representative cornering condition demands an accurate description of suspension and chassis K&C, as these directly impact the load transfer between the wheels and therefore the aerodynamic ride heights. It is also important to account for the nonlinear behavior of the tyres under

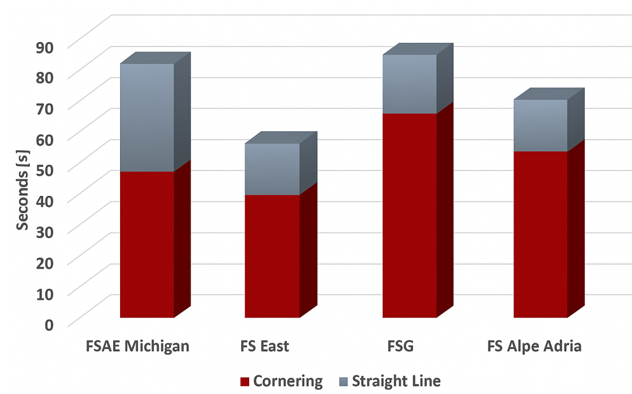


Figure 1: Time spent cornering on various 2023 competitions tracks

combined lateral and longitudinal forces, as well as the dynamic camber and toe changes resulting from suspension kinematics. The interaction between the suspension system and the aerodynamic loads should be modeled in a coupled manner, since the change in ride height will in turn influence the aerodynamic performance which can then further modify the vehicle's lateral balance. The lateral acceleration generated during cornering significantly alters the effective angle of attack on the wings and underbody devices, resulting in an asymmetric downforce distribution that is highly sensitive to even small perturbations in flow conditions. In addition, the induced side-slip angle and yaw rate of the vehicle introduce cross-flow components that require careful consideration of the boundary layer behavior and its impact on overall aerodynamic efficiency.

Such a high fidelity is both hard to achieve and unnecessary, the case will greatly benefit from using suitable hypothesis to simplify the models

To enable robust design iteration, the present study adopts a steady RANS formulation with $k-\omega$ SST and low-y^+ wall treatment. Tyre, suspension and chassis compliances are neglected and the operating point is assumed constant (speed, radius, steering). Transient cornering phenomena (turn-in/exit, vortex hysteresis) are outside scope and reserved for future work.

Modeling of the Cornering Case

The case we will be basing this model on will be a standard steady-state cornering situation, similar to that of the Skidpad Event commonly found is FSAE and Formula Student competitions.

We will assume the car is already in an equilibrium case, meaning that there is no variation of the pitch angle (no longitudinal acceleration), no variation of the steering input and constant load case at any point of the system.

Vehicle Dynamics Model

Modeling vehicle dynamics reliably is complex and difficult, often requiring a large number, up to 10 [3], 15 [4] or 27 [5], Degrees of Freedom (DoF). The mathematical complexity of such a system [6] can however be simplified. As we're less interested by a full, comprehensive, vehicle dynamics model, we will stick to a 2 DoF bicycle model. A 2-DoF bicycle model is a simplified representation of a vehicle in which the four wheels are reduced to two virtual wheels aligned along the vehicle's longitudinal axis. This model aims to output an approached behavior of the car, i.e its side-slip angle, steering angle and roll angle. A common representation of what we're trying to model is the following :

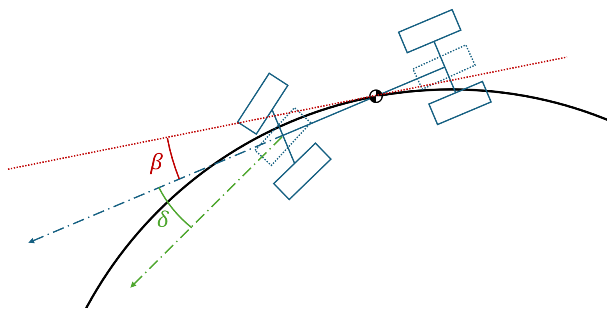


Figure 2: Top-View of the Cornering Case

This simplified view omits tyre slip-angles that aren't useful for the CFD model, but enter in sideslip and steering angle calculations. The wheels simplification of the bicycle model are dotted for reference. In order to find δ and β, we need to take an interest into tyre modeling to determine how much the car will slip under given conditions.

In order to generate the necessary data for cornering simulation, we designed a Python script that will extract and compile tyre data before computing several vehicle dynamics fundamentals equations[7], resulting in a simple vehicle dynamics model.

The details on the concerned parameters, their values and/or units is summed up at the end of this section in the Table 1 : *Parameters used in Vehicle Dynamics Modeling*.

The main inputs of this model are the cornering conditions of the car (total F_z, lateral acceleration, cornering radius) and the car's tyre model. The tyre model was based on experimental data extracted from round 9 of Tyre Test Consortium (TTC) regarding the Hoosier 16x7.5-10 R20 tyre. Those data allowed for an extraction of a $F_y = f(\alpha, F_z)$ function based on the Magic Formula of Hans B. Pacejka [8]. The 4-coefficients Magic Formula is of the form

$$F_y(\alpha) = D \cdot \sin[C \cdot \arctan(B\alpha - E(B\alpha - \arctan(B\alpha)))]$$

Where B, C, D, E are constants for a unique F_z case.

Those constants can then be fitted to account for F_z variations and produce a comprehensive model that will output the slip angle of a tyre depending on its lateral and vertical loads. The vertical load depends on the mass of the vehicle and the aerodynamic loads (i.e downforce) exerted on the car, while lateral loads only depends on lateral acceleration. From that, we can perform a regression on the B, C, D and E coefficients of the Magic Formula so they become dependent of F_z and can be generalized for further use :

- $B(F_z) = 3.79E^{-5} \cdot F_z^2 - 5.75E^{-2} \cdot F_z + 30.09$

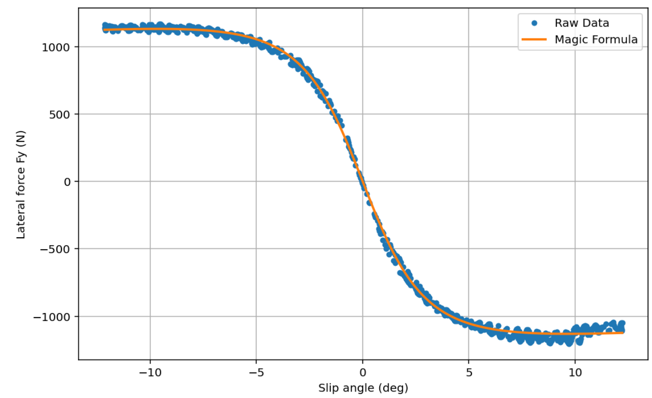


Figure 3: Pacejka Magic Formula curve fit to raw TTC data

- $C(F_z) = 1.03E^{-3} \cdot F_z + 1.05$
- $D(F_z) = -2.66 \cdot F_z - 55.48$
- $E(F_z) = -0.05 \cdot F_z + 0.78$

Those F_z-dependent coefficients can now be used to precisely compute the slip angles (or lateral forces), sideslip and steering angles for any F_z load case.

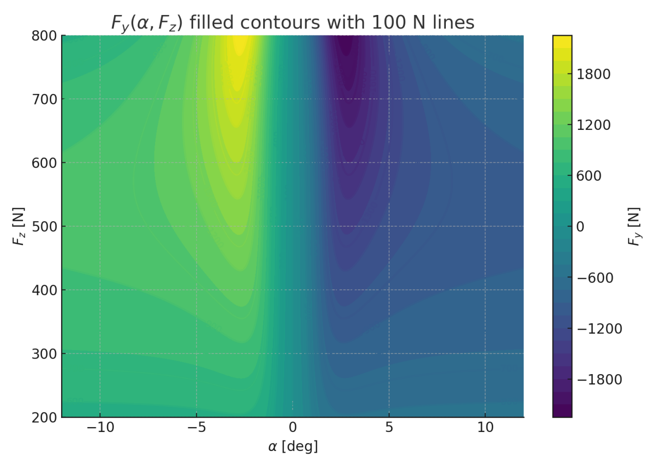


Figure 4: Heatmap for Fy against Fz and Slip Angle

For a bicycle model, the slip angles from the front and rear axles are tied to the steering angle δ by a formulation found in *Fundamentals of Vehicle Dynamics* by Thomas D. Gillespie[7] :

$$\delta_c = 57.3 \cdot \frac{W_B}{R} + \alpha_F - \alpha_R$$

Where δ_c is the median steering angle for a bicycle model. We then apply the anti-Ackermann suspension geometry parameters to determine δ_{in} and δ_{out} respectively for the steering angle of the inside and outside wheels.

The sideslip angle (or steady-state cornering yaw angle) relation to slip angles and steering angle can also be found in Gillepsie's book as :

$$\beta = \psi = 57.3 \cdot \frac{c}{R} - \alpha_R$$

This model also computes the rotational speed of each separate wheel of the car depending on the cornering radius and tangential speed (or lateral acceleration). The equations leading to those results will later be re-implemented in StarCCM+ for ease of use. The rotational speeds of the 4 wheels are given by :

$$\Omega_{FL} = \frac{\sqrt{a_c\, g}}{W_R}\left[\left(W_B(1-CG_x)+R\sin\beta\right)^2+\left(\frac{W_T}{2}-R\cos\beta\right)^2\right]^{1/4}$$

$$\Omega_{FR} = \frac{\sqrt{a_c\, g}}{W_R}\left[\left(W_B(1-CG_x)+R\sin\beta\right)^2+\left(\frac{W_T}{2}+R\cos\beta\right)^2\right]^{1/4}$$

$$\Omega_{RL} = \frac{\sqrt{a_c\, g}}{W_R}\left[\left(-W_B\, CG_x+R\sin\beta\right)^2+\left(\frac{W_T}{2}-R\cos\beta\right)^2\right]^{1/4}$$

$$\Omega_{RR} = \frac{\sqrt{a_c\, g}}{W_R}\left[\left(-W_B\, CG_x+R\sin\beta\right)^2+\left(\frac{W_T}{2}+R\cos\beta\right)^2\right]^{1/4}$$

Table 1: Parameters used in Vehicle Dynamics Modeling

X	Description	Value	Unit
Inputs			
g	Gravitational Acceleration	9.81	$[m.s^{-2}]$
a_c	Desired Lateral Acceleration		[G]
R	Cornering Radius		[m]
V	Tangential velocity [a]		[m/s]
m_{tot}	Car Mass (Weight + Aero)		[kg]
CG_x	Mass Repartition (X-axis)	0.4879	∅
W_B	Car Wheelbase	1525	[mm]
W_R	Radius of the Wheel	0.2032	[m]
W_T	Car Track	1123	[mm]
ϕ_{sens}	Roll Sensitivity	0.7	[deg/G]
Outputs			
δ_X [b]	Steering angle		[deg]
Ω_{XX}	Rotational speed of XX wheel		[rad/s]
ψ	Yaw Angle of the car [a]		[deg]
ϕ	Roll Angle of the car [c]		[deg]

[a] Applied to the car's center of mass
[b] Where c : center; ins : inside; out : outside
[c] Applied to the car's roll center

At this point in the modeling, the longitudinal forces on the tyre were neglected for simpler implementation, which proves to be a big hypothesis for the front axle due to steering angle offsets. Similarly, the use of a bicycle model neglects all weight transfer effects which will impact maximum achievable lateral acceleration. In order to stay within physical cases, the maximum lateral acceleration was limited to a on-track-measured maximum of 2.0Gs.

Although very simple, this model proves reliable physical coherence and slip angle values within a degree of precision against on-track slip angle measurements, which will be enough for our vehicle dynamics model. This model will likely be the subject of further work to create a precise comprehensive lap-time simulation to further validate the cornering situation datasets.

For our cornering test case, we will take inspiration from the Skidpad Event and impose a cornering radius of 8.125 meters and a lateral acceleration of 1.52Gs (tangential speed of 11 meters per second). The mass being slightly off a 50% repartition, the loads exerted on the front and rear axles are slightly different (see Figure 5).

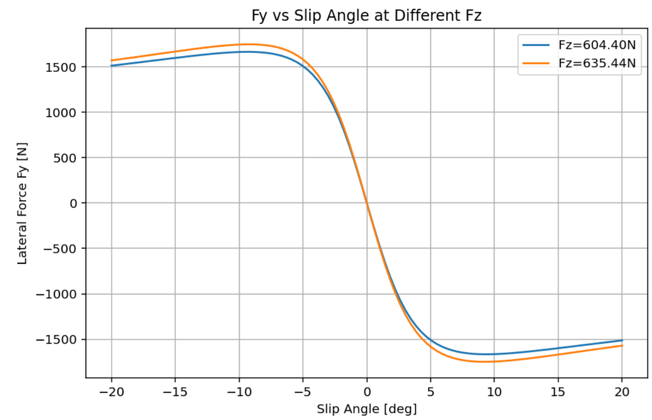


Figure 5: Loads on a tyre of the front (blue) and rear (orange) axle

The Python code gives us the set of output data that we will then use in the StarCCM+ Parameterized Model for the cornering test case situation :

Table 2: Output parameters for the R8,125-A1,52 Testcase

X	Description	Value	Unit
δ_{ins}	Inside Steering Angle	10.02	[deg]
δ_{out}	Inside Steering Angle	11.45	[deg]
$\psi = \beta$	Yaw Angle of the car	3.2279	[deg]
ϕ	Roll Angle of the car	1.0627	[deg]

Note : the model responds pretty well to extreme cases, and can be switched to a speed definition instead of an acceleration definition for high-speed/low-Gs cornering situations. In particular, it will truncate automatically lateral forces values that exceed the capacity of the tyre and display a warning that the car's stability is compromised.

Aerodynamics Model

From forces, coefficients and moments to balance and aeromaps, the study of aerodynamics is also a complex task that can improve or decrease performance. On previous work, we were able to characterize straight line aerodynamics via CFD models, on-track validation and wind tunnel methodologies. Those models are currently in-use in Formule ETS as part of their aerodynamics design department. The goal is now to tune the aforementioned vehicle dynamics model into a new CFD model, using unique modeling techniques [9][10][11] to study cornering aerodynamics.

Geometry and Setup

The first step in modeling this situation is positioning the car in the domain. This is done by creating a set of coordinate systems that will be used to modify the angles and position of a few elements :

- Two coordinate systems that will handle the Z-axis rotation of the front wheels following the input steering angles δ_{ins} and δ_{out}. This coordinate system is also used to define the rolling of the front wheels during the simulation.
- A "center of mass" coordinate system where we will apply the yaw rotation along the Z-axis to account for the sideslip β.
- A roll center, right below the rear axle of the car, to account for the X-wise rotation of the car, namely ψ.
- The last coordinate system will handle the Y-axis rotation of the rear wheels during the simulation.

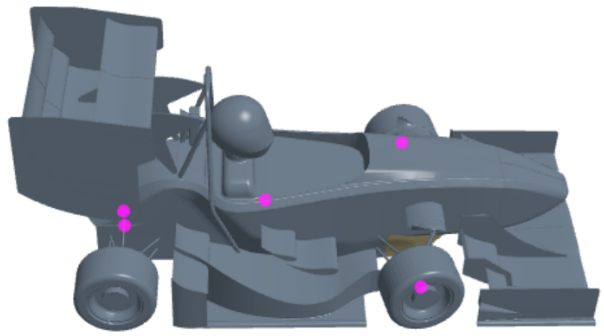


Figure 6: Coordinate Systems' centers positions

The domain is a 240 degrees "donut", with an air inlet (blue) and an outlet (orange). Wider and higher angles domain were tested but didn't prove significant improvement against the increase in computational time. The domain is entirely parameterized and dependent on the radius length. All domain walls are slip walls, except for the ground surface which is a no-slip wall. The car's parts are defined as no-slip walls.

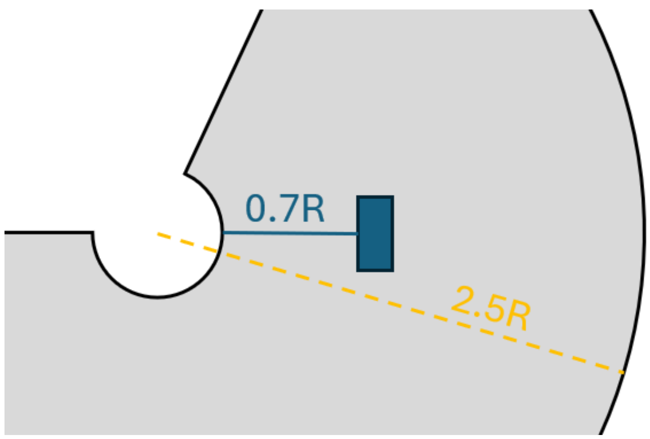
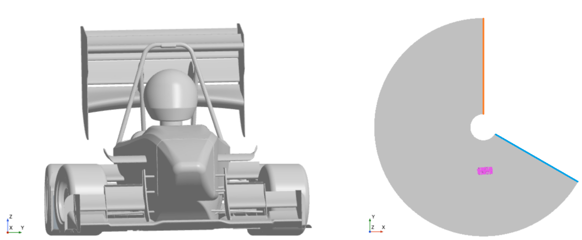


Figure 7: Definition of the domain regarding R

The necessary rotations and modifications are automatically applied before the meshing operations. Wheel rotation rates are also computed for boundary condition purposes. The Laboratory coordinate system is concentric to the center of rotation of the car. The car's center of mass is directly below the center of rotation at a distance of R meters.



(a) Front View - MANIC-24 (b) Top View - Domain

Figure 8: Final position of the car in the domain

In a similar way, we defined radius-dependent refinement boxes used for meshing that automatically updates with the radius parameter. There are 3 refinement boxes for wake management, and a few element-bound refinement boxes to improve detail and quality of difficult to mesh elements.

Mesh Generation

Aside from the different refinement boxes due to expectations of a curved wake, the mesh parameters are very similar to that of a SLSM case. The reader is referred to the reference [2] paper for details.

Equations & Solvers

In a similar way as for Mesh Generation, the main equations for the CFD model are the same to that of the SLSM [2] case :

- Reynolds-Averaged Navier-Stokes (RANS) equations model, adapted for a fast yet reliable computation.
- Steady-State time independence.
- $k-\omega$ SST closure model for turbulence approximation, coupled to low y^+ wall functions to ease convergence.
- Coupled solver algorithm, with automatic CFL detection and fixed under-relaxation factors.

The main difference with SLSM here lies in the fact that we use a Rotating Reference Frame (RRF). A RRF models steady, constant-radius cornering without mesh motion by solving the external-aerodynamics flow in a frame spinning about the vertical axis through the instantaneous center of rotation. The non-inertial formulation augments the momentum equation with Coriolis and centrifugal source terms, allowing the solver to reproduce curvature-induced lateral pressure gradients and curved streamlines under the floor and through the diffuser.

$$\underbrace{-2\,\rho\,\boldsymbol{\Omega}\times\mathbf{v}_r}_{\text{Coriolis}} \;-\; \underbrace{\rho\,\boldsymbol{\Omega}\times\big(\boldsymbol{\Omega}\times\mathbf{r}\big)}_{\text{centrifugal}} \;-\; \underbrace{\rho\,\dot{\boldsymbol{\Omega}}\times\mathbf{r}}_{\text{Euler (zero for steady }\Omega)}$$

Although there isn't any defined entry flows through the inlet or the outlet of the domain, boundary kinematics are applied via the relative-velocity field at inlets, the spatially varying moving ground, and local wheel MRF zones. The initial velocity field is radius-dependent and of the form

$$U = \Omega \cdot R$$

As such, since we're modeling a car moving through air, it is necessary to study a relative velocity field that fixes the rotation to get back to a usual CFD standard with air moving around a fixed object.

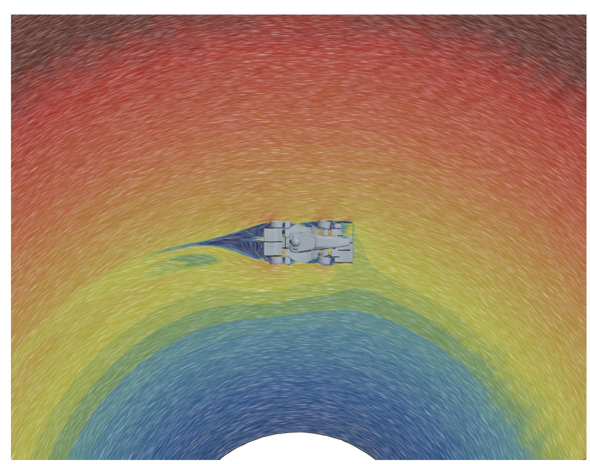


Figure 9: Example of a relative velocity field in a cornering situation

Results & Analysis

Formule ETS' 2025 prototype, DELTA-25, was studied for speeds comprised between 4 meters per second and 32 meters per second. Ride heights for front and rear axle are within the 15 millimeters to 40 millimeters range.

Changes in Side Force

Shift of the Pressure Center

Changes in wake

Validation

Convergence of the Solution

Mesh and Domain Independence

Wind-Tunnel Correlation - Yaw Sensitivity

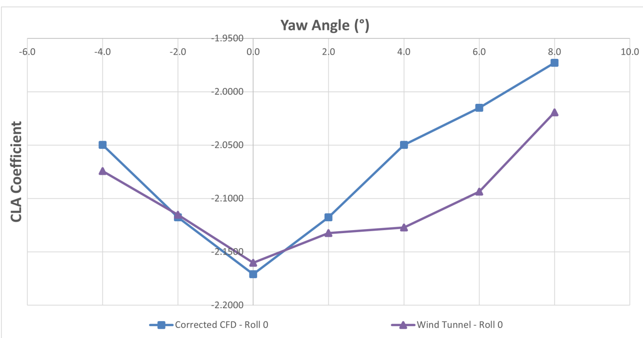


Figure 10: Example

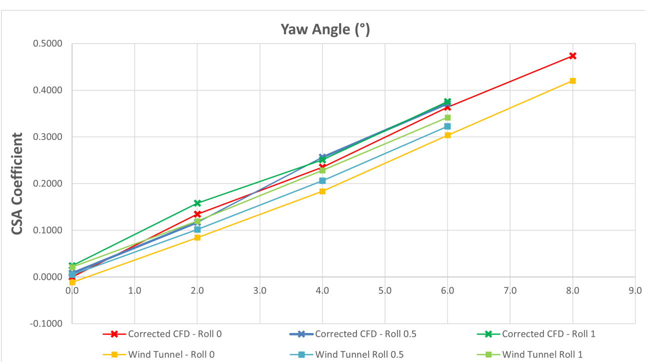


Figure 11: Example

Cross-validation with SLSM

Since the SLSM model was rigorously validated through both wind tunnel and on-track testing, it represents a reliable and verified way of predicting aerodynamical quantities such as CLA and CDA. By approaching straight line conditions with the cornering model (high cornering radius, low lateral acceleration, neutral steering) we can compare the results of both models and conclude on the CSM's reliability. By setting a radius of 250 meters, at a velocity of 20m/s, and comparing with a 20m/s SLSM case we obtain the following table.

The differences for CLA and CDA quantities fall within the 2-3% acceptable range, validating the methodology for the cornering simulation and authorizing the hypothesis of a CSM functioning at lower radii. Unfortunately, the aerobalance quantity shows a higher discrepancy from the SLSM model although proving to be closer from wind tunnel testing results.

	Dynamics Parameters			CFD Comparison		
	Rad. (m)	Vel. (m/s)	Lat. Gs (m/s^2)	CLA %diff	CDA %diff	AB-X %diff
SLSM	∞	20	0	0	0	0
CSM	250	20	0.1631	-2.81%	-3.73%	6.39%

Table 3: Differences in key quantities between SLSM and CSM

Conclusion

Throughout this project we created a reliable and efficient way of modeling cornering aerodynamics in a Formule SAE setting. The models we created proved consistence and precision within 5 to 10% against on-track data and CFD cross-referencing. We believe those tools will help understand and optimize aerodynamic design while increasing the relevancy of aerodynamics and the cornering performance without relying on technologies such as powered ground effect floors.

Recommendation

We recommend future work focus on transient cornering situations, accounting for transient vehicle dynamics and longitudinal forces, pitch angles. This will necessitate a more precise vehicle dynamics model accounting for a 4-wheeled vehicle under load-transfer conditions.

Although it might seem unnecessary, we also recommend focusing on CFD transient simulation of a full corner situation, including braking, coasting and re-acceleration. This will allow for a better understanding of the involved mechanics and will provide more complete data that is easier to correlate to track conditions.

Finally, as to further improve validation of the CSM model, wind tunnel fidelity to cornering situation could potentially be increased by creating a methodology allowing for wheel steering within the wind tunnel's test section. Moreover, tri-dimensionality of the wheel wake replication devices should be improved to further analyze steered wheel wake impact on aerodynamic devices.

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Definitions, Acronyms, Abbreviations

CFD	Computational Fluid Dynamics
CSM	Cornering Simulation Model
DoF	Degree of Freedom
FSAE	Formula Society of Automotive Engineers
K&C	Kinematics and Compliance
RANS	Reynolds-averaged Navier–Stokes
RRF	Rotating Reference Frame
SLSM	Straight Line Simulation Model
TTC	Tyre Test Consortium

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